Friday, February 8, 2007
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Math 53 — Section 108/109

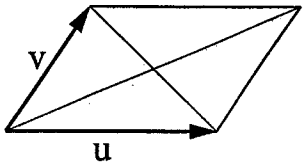
2. Quiz

Name _____

Time is 30 minutes. Do not use arguments from elementary geometry. Use vectors and dot-products. Please mark the answers by drawing a box around them.

1. Compute the angle between the diagonal and an edge of a cube. Hint: the unit cube is spanned by i, j and k. (Table below has some values of cosine)

α	$\cos(\alpha)$	α	$\cos(\alpha)$
60°	$\frac{1}{2}$	45°	$\frac{1}{\sqrt{2}}$
54.7°	$\frac{1}{\sqrt{3}}$	70.5°	$\frac{1}{3}$
75.5°	$\frac{1}{4}$	51.8°	$\frac{\sqrt{5}-1}{2}$
73.0°	$1-\sqrt{\frac{1}{2}}$	64.8°	$1-\sqrt{\frac{1}{3}}$

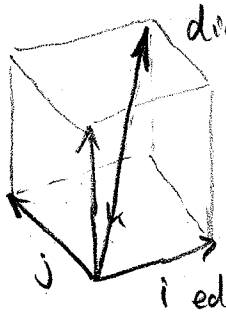


2. Show that the parallelogram spanned by the vectors **u** and **v** is a rectangle if the diagonals have equal length. See figure above.

3. Let $A(0,0)$, $B(9,-3)$ and $C(9,1)$. Give the coordinates of the point M which is closest to C and on the line through A and B.

Good Luck!

1



diagonal $d = i+j+k$

$$\cos\alpha = \frac{d \cdot i}{|d|\,|i|} = \frac{1}{\sqrt{3}\cdot 1}$$

$$\boxed{\alpha = 54.7°}$$

2

$u+v$, $u-v$ } diagonals

$$|u+v| = |u-v|$$

$$\Downarrow$$

$$(u+v)^2 = (u-v)^2$$

$$u^2 + 2uv + v^2 = u^2 - 2uv + v^2$$

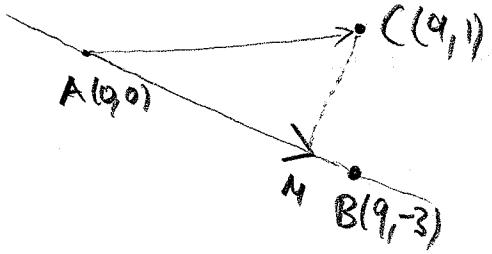
$$uv = -uv$$

$$\boxed{uv = 0}$$

$$\Downarrow$$

u perpendicular v, so it is a rectangle

3



$\vec{AM}$ is orthogonal projection of $\vec{AC}$ onto $\vec{AB}$

$\vec{AC} = \langle 9,1 \rangle$ $\quad$ $\vec{AB} = \langle 9,-3 \rangle$

$$\vec{AM} = \frac{\langle 9,1\rangle \cdot \langle 9,-3\rangle}{|\langle 9,-3\rangle|} \cdot \frac{\langle 9,-3\rangle}{|\langle 9,-3\rangle|}$$

$|\langle 9,-3\rangle| = \sqrt{9^2+(-3)^2}$
$= \sqrt{90}$

$$= \frac{(81-3)\langle 9,-3\rangle}{90}$$

$$= \frac{78\langle 9,-3\rangle}{90} = \frac{26\langle 3,-1\rangle}{10} = \frac{13\langle 3,-1\rangle}{5}$$

$$= \left\langle \frac{39}{5}, -\frac{13}{5} \right\rangle$$

$$\boxed{M = \left(\frac{39}{5}, -\frac{13}{5}\right)}$$

Check: $\vec{MC} = \left\langle 9 - \frac{39}{5}, 1 + \frac{13}{5} \right\rangle = \left\langle \frac{6}{5}, \frac{18}{5} \right\rangle$

$\vec{AB} \cdot \vec{MC} = 0$ ✓