Math53, Spring 2008
Section 108/109

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Divergence and Stokes Theorem

1. Let S be a closed surface. Use the divergence theorem to show that

$$\iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S} = 0.$$

2. Let S be the sphere around $(0,0,1)$ with radius 1 and outward orientation. Let S' be the part of S lying above the plane $z = z_0$. As $z_0 \to 0$, notice that

$$\iint_{S'} \text{curl}\mathbf{F} \cdot d\mathbf{S} \to \iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S}.$$

   Use Stoke's Theorem to show that

$$\iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S} = 0.$$

3. Let f and g (continuous) functions such that for all regions E

$$\iiint_E f dV = \iiint_E g dV.$$

   What can you say about f and g?

4. Let $f(\mathbf{r}, t)$ and $g(\mathbf{r}, t)$ be (differentiable) functions such that for all regions E

$$\frac{d}{dt} \iiint_E f dV = \iiint_E g dV.$$

   What can you say about f and g?

5. Let $\mathbf{F}(x, y, z) = y\mathbf{i}$ and S be the surface of a box $[-1, +1]^3$ without top part and outward orientation. Compute

$$\iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S}$$

   in two different ways using the definition of surface integrals and using Stoke's theorem.