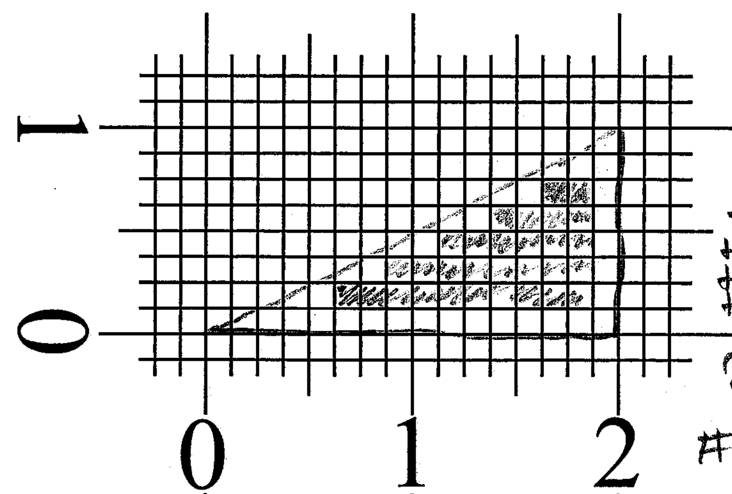
No books, notes or calculators are allowed. To receive full credit you must show all of your work. You have 15 *minutes to complete the quiz.*

Name (Last, First) and UID: ______________________

1. Using Lagrange multipliers, find all potential extrema of $f(x,y,z) = 3x^3 + 4y^3$ on the surface given by $g(x,y,z) = x^2 + y^2 + z^2 - 1 = 0$. Hint: Distinguish the case where $z \neq 0$ and $z = 0$. For $z = 0$, distinguish the cases where $x = 0$, $y = 0$, and both $x, y \neq 0$.

2. Let T be the open triangle with vertices $(0,0), (2,0), (2,1)$. Let $\chi_T : \mathbb{R}^2 \to \mathbb{R}$ be the characteristic function of T defined by $\chi_T(x) = 1$ if x in T and $\chi_T(x) = 0$ otherwise. Let P_k be the standard partition of $\mathbb{R}^2$ consisting of rectangles $[x_1 2^{-k}, (x_1+1)2^{-k}] \times [x_2 2^{-k}, (x_2+1)2^{-k}]$ with x_1, x_2 integers. For $k = 3$, draw a picture of U in the grids below and fill all the rectangles in P_k that are contained in T (top), and of all rectangles in P_k that are not disjoint with T (bottom). Give a formula for the lower and upper Darboux sum $L(\chi_T, P_k)$ and $U(\chi_T, P_k)$ in terms of $k > 0$. Recall that $1 + 2 + 3 + \cdots + n = n(n+1)/2$. Show that the lower and upper Darboux sum converge to the same value as $k \to \infty$. What is that value?



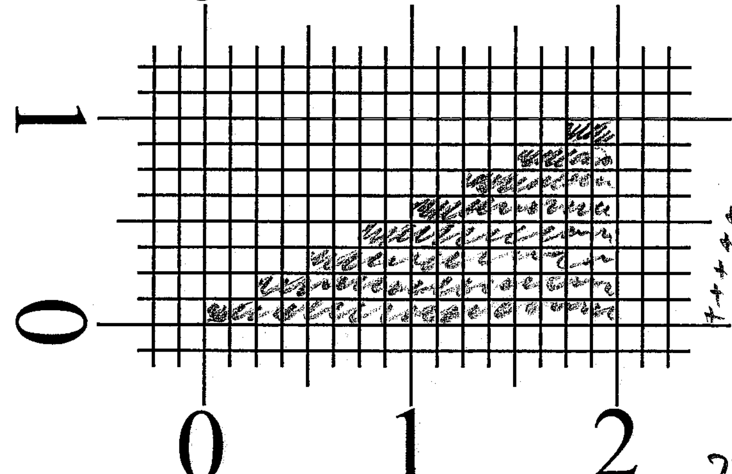
$2 + 4 + 6 + 8 + 10$

In general:

$2 + 4 + \cdots + 2\cdot(2^k - 3)$

$\#\text{Rectangles} = 2\cdot\frac{(2^k-3)(2^k-2)}{2}$

$= 4^k - 5\cdot 2^k + 6$



$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16$

In general:

$2 + 4 + \cdots + 2\cdot 2^k$

$\#\text{Rectangles} = 2\cdot\frac{2^k(2^k+1)}{2}$

$= 4^k + 2^k$

1.)

$$\nabla f = \begin{pmatrix} 9x^2 \\ 12y^2 \\ 0 \end{pmatrix} \qquad \nabla g = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

$$\nabla f = \lambda \nabla g \qquad \begin{pmatrix} 9x^2 \\ 12y^2 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

If $z \neq 0, \lambda = 0, \Rightarrow x, y = 0$, $\boxed{\pm\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$ is potential extremum.

If $z = 0$ and $x = 0$, then $y = \pm 1$, so potential extrema at

$$\boxed{\pm\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}.$$

If $z = 0$ and $y = 0$, then $x = \pm 1$, so potential at extrema at

$$\boxed{\pm\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}$$

If $z = 0$, $x, y \neq 0$, then $9x^2 = \lambda 2x$, $12y^2 = \lambda 2y$.

$x = 2\lambda/9 \quad y = 2\lambda/12$, so solution of form

$\frac{2\lambda}{36}\begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}$, $\left\|\begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}\right\| = 5$, so potential extrema

at $\boxed{\pm\begin{pmatrix} 4/5 \\ 3/5 \\ 0 \end{pmatrix}}$.

2)

$$L(\chi_T, P_k) = 2^{-2\cdot k} \cdot (4^k - 5\cdot 2^k + 6)$$

$$= 1 - 5\cdot 2^{-k} + 6\cdot 2^{-2k} \longrightarrow 1$$

$$U(\chi_T, P_k) = 2^{-2\cdot k} (4^k + 2^k)$$

$$= 1 + 2^{-k} \longrightarrow 1$$

$$\int \chi_T = 1$$