1.1

Affine approximation at 0: $x+z=0$ so it has <u>no</u> singularity at 0

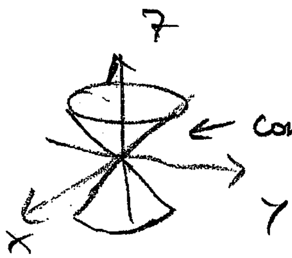
tangent plane at 0 given by $z=-x$

So $Df = \begin{pmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

1.2 $g(x,y,z) = x^2+y^2-z^2$ has $\nabla g = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$, so $\nabla g(0)=0$

has singularity at 0.

<u>Picture</u>



← cone has singularity at 0

1.3 $g(x,y,z) = (x-1)^2+y^2-z^2-1$ has $\nabla g = \begin{pmatrix} 2(x-1) \\ 2y \\ 2z \end{pmatrix}$ so at $\nabla g(0) = \begin{pmatrix} -2 \\ 0 \\ \boxed{0} \end{pmatrix}$

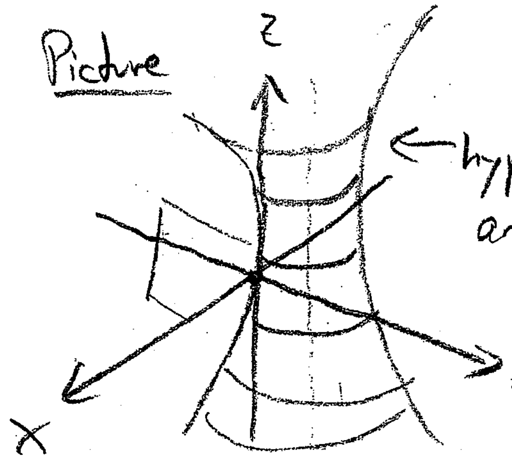
so <u>no</u> singularity at 0,

but $\frac{\partial g}{\partial z}(0) = 0$, so

<u>not</u> graph $z = f(x,y)$.

unless you permute x,y,z

<u>Picture</u>



← hyperboloid around point $(1,0,0)$ touching origin,

tangent plane at 0 is y-z-plane

1.4 $(x-1)^2 + y^2 - z^2 = 0$ not zero at O,
so subspace empty near zero, no singularity at O.

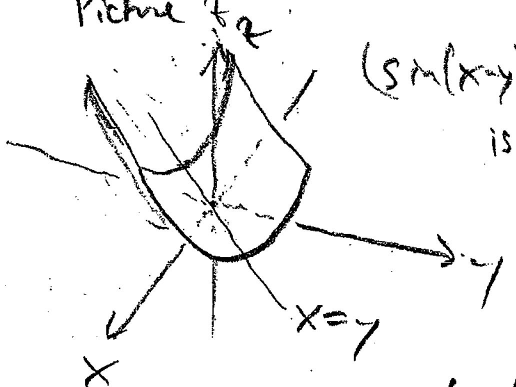
2.1 Affine approximation at O:

$-z = 0, \quad z = 0$, so

$\nabla f = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad \nabla g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ not linearly independent.

$\Rightarrow$ singularity at O

Picture



$(\sin(x-y))^2 - z = 0$
is approx. an extruded parabel pointing up
touching x-y plane
at $x=y$ line

$(\sin(x+y))^2 + z$ is similar, pointing down
touching x-y plane at
$x=-y$ line,
intersect only in the point $(0,0,0)$
near O.

2.2

Affine approximations at 0:

$$-2x + 2y - z = 0 \qquad \nabla f(0) = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$$

$$-2x - 2y + z = 0 \qquad \nabla g(0) = \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}$$

$$\nabla f(0) \times \nabla g(0) = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+2 \\ 2-2 \\ +4+4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ +8 \end{pmatrix}$$

So no singularity, tangent vector is multiple of $\begin{pmatrix} 4 \\ 0 \\ +8 \end{pmatrix}$