Wednesday, Feburary 29, 2012 | Quiz 3 | Advanced Calculus MATH 411 Section 0201

This quiz consists of 3 questions. No books, notes or calculators are allowed. To receive full credit you must show all of your work. You have 15 minutes to complete the quiz.

Name (Last, First) and UID: _____

1. What are the following limits (with proof)?

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^4+y^4)}{x^4+y^4} \quad \text{and} \quad \lim_{(x,y)\to(0,0)} \frac{x^3y^3}{x^2+y^2}$$

$t = x^4+y^4 \quad t\to 0 \text{ as } (x,y)\to 0$ so

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^4+y^4)}{x^4+y^4} = \lim_{t\to 0} \frac{\sin(t)}{t} = \lim_{t\to 0} \frac{\cos(t)}{1} = 1$$

$$\left|\frac{x^3y^3}{x^2+y^2}\right| \le |xy|\left|\frac{x^2}{x^2+y^2}\right|\left|\frac{y^2}{x^2+y^2}\right| \le |xy| \to 0 \text{ as } (x,y)\to(0,0), \text{ so } \lim \frac{x^3y^3}{x^2+y^2} = 0$$

2. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function such that $\lim_{\mathbf{x}\to 0} \frac{f(\mathbf{x})}{\|\mathbf{x}\|} = 0$. For each of the following two statements, give either a counter example or proof: $\lim_{\mathbf{x}\to 0} f(\mathbf{x}) = 0$ and $\lim_{\mathbf{x}\to 0} \frac{f(\mathbf{x})}{\|\mathbf{x}\|^2} = 0$.

$$\left.\begin{array}{l}\lim_{x\to 0} \frac{f(x)}{\|x\|} = 0 \\ \lim_{x\to 0} \|x\| = 0\end{array}\right\} \Rightarrow \lim_{x\to 0} \frac{f(x)}{\|x\|}\|x\| = 0$$

$$\lim_{x\to 0} f(x) = 0 \ \checkmark$$

$f(x) = \|x\|^2$

and $\lim_{x\to 0} \frac{f(x)}{\|x\|^2} = 1 \neq 0$

Counter example.

3. Let $f(x,y,z) = x + xy + yz + x^2\sin(xz)$. Compute $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial y^2}$.

$$\frac{\partial f}{\partial x} = 1 + y + 2x\sin(xz) + x^2 z\cos(xz) \qquad \frac{\partial f}{\partial y} = x + z$$

$$\frac{\partial^2 f}{\partial x^2} = 2\sin(xz) + 2xz\cos(xz) + 2xz\cos(xz) - x^2z^2\sin(xz)$$

$$= 2\sin(xz) + 4xz\cos(xz) - x^2z^2\sin(xz)$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 1$$

$$\frac{\partial^2 f}{\partial y^2} = 0$$