Wednesday, Feburary 15, 2012 | Quiz 2 | Advanced Calculus MATH 411 Section 0201

*This quiz consists of* 3 *questions. No books, notes or calculators are allowed. To receive full credit you must show all of your work. You have* 15 *minutes to complete the quiz.*

**Name (Last, First) and UID:** ____________________

1. Let $f$ be continuous, but not linear. Which of the following sets is connected, pathwise-connected, respectively, convex? No proof necessary. 45%

each 5%

| | connected | pathwise-connected | convex |
|---|---|---|---|
| $\{(x,y) \in \mathbb{R}^2$ such that $x$ or $y$ is rational$\}$ | ☑ | ☑ | ☐ |
| $\{(x,y) \in \mathbb{R}^2$ such that $x$ or $y$ is an integer$\}$ | ☑ | ☑ | ☐ |
| $\{(x,y) \in \mathbb{R}^2$ such that $x$ and $y$ are rational$\}$ | ☐ | ☐ | ☐ |
| $\{(x,y) \in \mathbb{R}^2$ such that $x^2+y^2=1\}$ | ☑ | ☑ | ☐ |
| $\{(x,y) \in \mathbb{R}^2$ such that $x^2+y^2<1\}$ | ☑ | ☑ | ☑ |
| $\{(x,y) \in \mathbb{R}^2$ such that $x^2>1\}$ | ☐ | ☐ | ☐ |
| $\{(x,y) \in \mathbb{R}^2$ such that $x$ is in $[-1,1]$ and $y=f(x)\}$ | ☑ | ☑ | ☐ |
| $\{(x,y) \in \mathbb{R}^2$ such that $x$ is not in $[-1,1]$ and $y=f(x)\}$ | ☐ | ☐ | ☐ |
| $\{(x,y) \in \mathbb{R}^2$ such that $1<x^2+y^2<4\}$ | ☑ | ☑ | ☐ |

2. Choose any of the above sets that is not connected, call it $A$. Now, prove that it is not connected by finding two open sets $U$ and $V$ that separate $A$, i.e., two open sets $U$ and $V$ such that $A$ is the union of $A \cap U$ and $A \cap V$ and such that $A \cap U$ and $A \cap V$ are disjoint. 25%

$U = \{(x,y) \in \mathbb{R}^2$ such that $x < 1/\sqrt{2}\}$
$V = \{(x,y) \in \mathbb{R}^2$ such that $x > 1/\sqrt{2}\}$
separates all three cases.

3. Let $u=(1,1,1)$. Let $A \subset \mathbb{R}^3$ be a connected subset containing the points $(-1,0,0)$ and $(1,0,0)$. Show that there is a point $v$ in $A$ such that $\langle u,v\rangle = 0$. 30%

$f: v \longmapsto \langle u,v \rangle$ is cont's in $v$.

$f(-1,0,0) = -1$
$f(1,0,0) = +1$
by Intermediate Value Theorem, there is $v \in A$ such that $f(v) = 0$.