Wednesday, Feburary 1, 2012 | Quiz 1 | Advanced Calculus MATH 411 Section 0201

This quiz consists of 3 *questions. No books, notes or calculators are allowed. To receive full credit you must show all of your work. You have* 15 *minutes to complete the quiz.*

Name (Last, First) and UID: _____

If you want your grades posted under an alias instead of the last four digits of UID, please write the alias. If you don't want your grades posted online, please write "NO": _____
How many hours did you work on the first homework assignment for this class? _____

30%

1. Let $u = (1, 2, 3)$, $v = (-8, 1, 2)$. Compute $||u||$, $||v||$, and show that u and v are orthogonal.

$||u|| = \sqrt{1^2+2^2+3^2} = \sqrt{1+4+9} = \sqrt{14}$

$||v|| = \sqrt{(-8)^2+1^2+2^2} = \sqrt{64+1+4} = \sqrt{69}$

$\langle u, v\rangle = 1\cdot(-8) + 1\cdot 2 + 3\cdot 2 = -8+2+6 = 0$

$\Rightarrow$ orthogonal

35%

2. Compute the angle between $(1, 1, -1)$ and $(\sqrt{6}, 1, 1)$ in degrees.

$u = \quad v =$

$||u|| = \sqrt{1^2+1^2+(-1)^2} = \sqrt{3}$ $\qquad ||v|| = \sqrt{\sqrt{6}^2+1^2+1^2} = \sqrt{6+1+1} = \sqrt{8} = 2\sqrt{2}$

$\langle u, v\rangle = 1\cdot\sqrt{6} + 1\cdot 1 + (-1)\cdot 1 = \sqrt{6}+1-1 = \sqrt{6}$

$\langle u, v\rangle = ||u||\,||v|| \cos\theta$ $\qquad \cos\theta = \dfrac{\langle u, v\rangle}{||u||\,||v||} = \dfrac{\sqrt{6}}{2\sqrt{2}\sqrt{3}} = \dfrac{1}{2}$

$\Rightarrow \theta = 60^\circ$ (15%)

35%

3. Use the Cauchy-Schwarz inequality to show that

$$\left|\frac{3x+4y}{\sqrt{x^2+y^2}}\right| \leq 5$$

$\langle u, v\rangle = 3x + 4y$

Let $u = (x, y)$ $\quad v = (3, 4)$. $\quad ||u|| = \sqrt{x^2+y^2}$ $\quad ||v|| = 5$

Cauchy-Schwarz inequality $|\langle u, v\rangle| \leq ||u||\,||v||$

$$\Rightarrow \left|\frac{\langle u, v\rangle}{||u||}\right| \leq ||v||$$

$$\Rightarrow \left|\frac{3x+4y}{\sqrt{x^2+y^2}}\right| \leq 5$$