1. Let $S = [0,1]^n$. Let $f : S \to \mathbb{R}$ be a function such that all partial derivatives exists and are bounded by M: $\left|\frac{\partial f}{\partial x_i}\right| < M$. Use the fundamental theorem of calculus to give an estimate for $|f(x + he_i) - f(x)|$ where e_i is the i-th basis vector. Let P_k be the standard partition by rectangles $[x_1 2^{-k}, (x_1+1)2^{-k}] \times \cdots \times [x_n 2^{-k}, (x_n+1)2^{-k}]$ where $(x_1, \ldots, x_n) \in \mathbb{Z}^n$. Given a rectangle $I \subset S$ in P_k, give an estimate for $|f(q) - f(q')|$ where q and q' are in I. Let $G = U(f, P_k) - L(f, P_k)$. Give an estimate of the contribution of those rectangles to G. Give an estimate for $|f(q)|$ where $q \in S$. Give an estimate of the volumes for the rectangles I in P_k not contained in S but bordering S. Now, give an estimate of the contribution of those rectangles to G. Give an estimate for G. Proof that f is Riemann-integrable.

2. Which of the following sets has Riemann-measure 0? Which Lebesgue-measure 0? Give a brief argument.

   1. $[0,1] \times \{0,1\} \subset \mathbb{R}^2$.
   2. $[0,1] \times [0,1] \times \{0,1\} \subset \mathbb{R}^3$.
   3. $\mathbb{R} \times \{0,1\} \subset \mathbb{R}^2$.
   4. $\{(x,y,z) | x^2 + y^2 + z^2 = 1\} \subset \mathbb{R}^3$.
   5. $\{(x,y,z) | x, y, \text{ and } z \text{ are in } \mathbb{Q}\} \subset \mathbb{R}^3$.
   6. $\{(x,y,z) | x \text{ is in } \mathbb{Q}\} \subset \mathbb{R}^3$.
   7. $\{(x,y,z) | x, y, \text{ or } z \text{ is in } \mathbb{Q}\} \subset \mathbb{R}^3$.